Improving AI Decision Modeling Through Utility Theory

Dave Mark
President & Lead Designer
Intrinsic Algorithm LLC

Kevin Dill
AI Engineer
Lockheed Martin
Dave Mark

- President & Lead Designer of Intrinsic Algorithm LLC, Omaha, NE
  - Independent Game Studio
  - AI Consulting Company

- Author of Behavioral Mathematics for Game AI
What's new in 2009 is:

1. There's now an agreed-upon name for this architecture: utility-based, which is much more reflective of how it works. Previous names, such as "Goal-Based Architectures" that Kevin Dill used were particularly overloaded already.

2. A group of developers advocate building entire architectures around utility, and not only sprinkling these old-school scoring-systems around your AI as you need them.

The second point is probably the most controversial.
We do requests…

“Wow… you’ve got a lot of stuff on utility modeling in here…
You should do a lecture on this stuff at the AI Summit.”

Daniel Kline
Outside P. F. Chang’s
Stanford Mall
October 2009
What is “Utility Theory”?  

In economics, utility is a measure of the relative satisfaction from, or desirability of, consumption of various goods and services. Given this measure, one may speak meaningfully of increasing or decreasing utility, and thereby explain economic behavior in terms of attempts to increase one's utility.
What is “Utility Theory”?  

• How much is something worth to me?  
• Not necessarily equal to “value”  
  – E.g. $20 might mean more or less than $20  
• Allows comparisons between concepts  
• Allows decision analyses between competing interests  
• “Maximization of expected utility”
What is “Utility Theory”?  

- Related to…
  - Game theory
  - Decision theory

- Used by…
  - Economics
  - Business
  - Psychology
  - Biology

John von Neumann
Value Allows Analysis

• Converting raw numbers to usable concepts
  – Distance
  – Ammo
  – Health

• Converting raw numbers to *useful* concepts
  – Distance → Threat
  – Ammo → Reload Necessity
  – Health → Heal Necessity
Value Allows Comparisons

- By assigning value to a selection, we can compare it to others
- Von Neumann and Morgenstern’s game theory
- Without value, comparisons are difficult… or even impossible!
Marginal Utility

- Utility isn’t always the same
Marginal Utility

- Decreasing Marginal Utility
  - Each additional unit is worth less than the one before
  - The rate of increase of the total utility decreases
  - Utility of 20 slices $\neq 20 \times$ Utility of 1 slice
Marginal Utility

- Increasing Marginal Utility
  - Each additional unit is worth more than the one before
  - The rate of increase of the total utility increases
  - Utility of 20 Lego $\neq$ 20 * Utility of 1 Lego
Converting Data to Concepts

• What does the information say?
• Raw data doesn’t mean much without context
• If data is ambiguous, we can’t reason on it
• Various techniques to make sense of raw data
  – Conversion formulas
  – Response curves
  – Normalization (e.g. 0..1)
As the distance changes, how much anxiety do you have?
Simple Rule

If distance <= 30 then anxiety = 1
Linear Formula

Anxiety = (100 – distance) / 100
Exponential Formula

Anxiety $= \frac{100 - \text{distance}^3}{100^3}$
Changing Exponents

Anxiety = (100 – distance\(^k\)) / (100\(^k\))
\(k = 2, 3, 4, 6\)
Shifting the Curve

Exponent Function Variations

Distance

Anxiety

0.00
0.25
0.50
0.75
1.00

0 1 0 2 0 3 0 4 0 5 0 6 0 7 0 8 0 9 1 0 0

0 1 0 2 0 3 0 4 0 5 0 6 0 7 0 8 0 9 1 0 0
Threshold / Linear / Exponential

Exponential Threshold

Distance

Anxiety

- Binary
- Linear
- Exponential

Distance
Logistic Function

(One of the sigmoid – or “s-shaped” – functions)

\[ y = \frac{1}{1 + e^{-x}} \]
Logistic Function

Anxiety = \frac{1}{1+(2.718 \times 0.45)^{\text{distance}+40}}
Variations on the Logistic Curve

Anxiety = \frac{1}{1 + (2.718 \times 0.45)^{\text{distance} + 40}}
Shifting the Logistic Function

Anxiety = $\frac{1}{1+(2.718 \times 0.45)^{\text{distance}+40}}$
Logit Function

\[ y = \log \frac{e^x}{1 - x} \]
Logit Function

\[ y = \log_e \left( \frac{x}{1-x} \right) \]
Logit Function

\[ y = \log \left( \frac{x}{1 - x} \right) \]
Logit Function

\[ y = \log_e\left(\frac{x}{1-x}\right) + 5 \]
Logit Function

\[ y = \frac{\log_e\left(\frac{x}{1-x}\right) + 5}{10} \]
How Do We Model Our Information?

• Increasing or Decreasing?

• Rates of change
  – Steady or Variable?
  – Inflection Point?

• Amount of change
  – Constrained or Infinite?
  – Asymptotic?
But What Good Is It?

When Anxiety > \( n \) then…

![Exponential Threshold Graph](image-url)
Comparing Apples and Ammo

- By using normalized utility values, we can define relationships and comparisons that otherwise would have been obscure
  - Risk vs. Reward (game theory)
  - Fear vs. Hate
  - Ammo vs. Health
Comparing Apples and Ammo

- 100 Health (Max)
- 75 Health
- 50 Health
- 25 Health (??)
- 5 Health (!!!!)

- 100 Ammo (Max)
- 75 Ammo
- 50 Ammo
- 25 Ammo
- 5 Ammo

Normalized Importance of Taking Action

Heal
Reload
Comparing Apples and Ammo

- As health decreases, urgency to heal increases
- Make sure we don’t get too low on health!

- As ammo decreases, urgency to reload increases
- Urgency hits maximum when we are out of ammo
Comparing Apples and Ammo

- Collect current states of independent variables
- Normalize using response curves
- (Combine as necessary)
- Compare normalized values and select:
  - Highest scoring selection
  - Weighted random from all choices
  - Weighted random from top \( n \) choices
Comparing Apples and Ammo

Normalized Importance of Taking Action

Enemy Strength

Threat Level

0.684

Threat
Comparing Apples and Ammo

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<th>Ammo</th>
<th>Health</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>Utility</td>
<td><strong>0.684</strong></td>
<td>0.118</td>
</tr>
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**Graphs: Normalized Importance of Taking Action**

1. **Threat Level** graph showing a normalized importance value of **0.684**.
2. **Heal** graph showing a normalized importance value of **0.118**.
3. **Reload** graph showing a normalized importance value of **0.125**.
Comparing Apples and Ammo

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<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Utility</td>
<td>0.684</td>
<td>0.729</td>
</tr>
</tbody>
</table>

Normalized Importance of Taking Action

- **Threat Level**
  - 0.684
  - Value: 85
  - Utility: 0.684

- **Value**
  - Importance: 0.729
  - Value: 0.729

- **Health**
  - Importance: 0.003
  - Value: 0.003
Comparing Apples and Ammo

<table>
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<th>Health</th>
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</thead>
<tbody>
<tr>
<td>Value</td>
<td>0</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>Utility</td>
<td>0.000</td>
<td>0.016</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Normalized Importance of Taking Action

Threat Level

- Threat

Normalized Importance of Taking Action

- Value

Importance

- Reload

- Heal

0.125

0.016
Comparing Apples and Ammo

- Don’t simply process 1 potential action at a time
  - Should I attack?
  - Should I reload?
  - Should I heal?
  - Should I have a beer?
- Compare all potential actions to each other
  - Of all of the things I could do, which is the most important at this moment?
Beyond Apples and Ammo

• Utility measurements can model more than simply tangible data
• They can model abstract concepts:
  – Threat
  – Safety
  – Morale
  – Emotions
Stacking Apples and Ammo

- Individual utility value can be combined to form new conceptual utilities
- “Need to take cover”
  - Amount of fire being taken (Threat)?
  - Is it almost time to reload?
  - Is it almost time to heal?

\[
\text{Cover} = (0.2 + \text{Reload} + (\text{Heal} \times 1.5)) \times (\text{Threat} \times 1.3)
\]
Stacking Apples and Ammo

Cover = (0.2 + Reload + (Heal x 1.5)) x (Threat x 1.3)
Stacking Apples and Ammo

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Relative Utilities

<table>
<thead>
<tr>
<th></th>
<th>0.400</th>
<th>0.300</th>
<th>0.100</th>
<th>0.137</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reload</td>
<td>0.250</td>
<td>0.500</td>
<td>0.750</td>
<td>1.000</td>
</tr>
<tr>
<td>Heal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threat</td>
<td></td>
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<td></td>
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</tr>
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Stacking Apples and Ammo

Cover = (0.2 + Reload + (Heal x 1.5)) x (Threat x 1.3)

Relative Utilities

<p>| | | | |</p>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.900</td>
<td>0.900</td>
<td>0.050</td>
</tr>
<tr>
<td>Reload</td>
<td>Heal</td>
<td>Threat</td>
<td>Cover</td>
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Normalized Utility
Utility of Time

• Time can be converted into a utility value
  – Time to travel over distance
  – Time to complete something
• Utility of time can be used for comparisons
• Utility of time can modify other utilities
Utility of Time

All other things being equal, select the closest goal
Utility of Time

How does the 2x difference in the relative utility of the goals compare to the 2x difference in the distances?
Utility of Time

By taking the keys out of consideration as a potential action, we neglect to get them as we pass right by them.
Utility of Time

- Normalized distance utility as inverse of distance
- Use as coefficient to modify base utility of getting keys
Utility of Time

By keeping the keys *in* consideration at all times, and factoring in the utility of time, we get them as we pass by them.
Stacking It All Up

- Number of Allies
- Strength of Allies
- Number of Enemies
- Strength of Enemies

- Allied Strength
- Enemy Strength

- Threat Ratio
- Proximity to Base

- Urgency

- My Health
- Proximity to Leader

- My Morale

- Retreat Score

“Compartmentalized Confidence”
Spreading It All Out

Number of Allies
Strength of Allies
Number of Enemies
Strength of Enemies

Threat Ratio
Allied Strength
Enemy Strength

Proximity to Leader
My Health

Proximity to Base
My Morale

Urgency

Retreat Score

Data processing != Decision processing
Managing Scalability

- Don’t perform all calculations every frame
  - Every $n$ frames
  - Use triggered updates
- Split data calculation off into separate processes
  - Used by multiple utility calculations for same agent
  - Used by decision calculations for multiple agents
  - Blackboard architecture to manage and store
- Lends itself well to multi-threading
Everything is Relative

• Many AI decision processes (BTs, FSMs):
  – Examine one choice at a time and ask “should I do this one thing?”
  – Are certain parameters met to justify that choice?
  – If not, move on to the next one in a pre-specified order

• What happens if no options meet their criteria?
  – Fall back (idle) behavior may not be appropriate
  – Very susceptible to edge cases
Everything is Relative

- Utility-based architectures:
  - Continuously analyze all options (rather than just one)
  - Rate all options based on their respective factors
  - Select the option that is most appropriate at the time
- Not based on arbitrary, independent thresholds
- Handles situational edge cases better
- Easier to manage potentially conflicting logic
Dave Mark
Intrinsic Algorithm LLC

Dave@IntrinsicAlgorithm.com

www.IntrinsicAlgorithm.com

(402) 208–7054

IADaveMark
on:
Yahoo – AIM – Skype – Twitter
Kevin Dill

- 9 year industry veteran
- Staff Software Engineer, Lockheed Martin
- Lecturer, Boston University
- Technical Editor, Charles River Media
**Example: Apartment Shopping**

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
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<tbody>
<tr>
<td>Beautiful loft apartment</td>
<td>Great view… of a used car lot</td>
</tr>
<tr>
<td>Convenient shopping district</td>
<td>No off-street parking</td>
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<td>Close to work</td>
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</tr>
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**606 Automobile Way, Apt 316**

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low rent</td>
<td>45 minute commute</td>
</tr>
<tr>
<td>Electricity &amp; water included</td>
<td>No shopping nearby</td>
</tr>
<tr>
<td>Beautiful wooded lot</td>
<td>Landlady lives upstairs</td>
</tr>
<tr>
<td>Nearby parks, bike trails</td>
<td></td>
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Key Insight

• We have 12 distinct pros and cons, but only 4 types of considerations:
  – Cost
  – Distance to __________
    • Could be the distance to work, shopping, parking, etc.
  – Aesthetic (i.e. how nice looking is the place)
    • Could be interior or exterior
    • Obviously, there is a lot of variability in what people consider to be “nice”
  – Noise restrictions
• Many of these are reusable in other contexts!!
### Example: Apartment Shopping

#### 606 Automobile Way, Apt 316

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#### 10-B Placid Avenue

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Executive Joe

• Joe is a high-powered executive at a big bank
• Joe makes lots of money
  – Cost doesn’t matter
• Joe works late most every night
  – Exterior aesthetics don’t matter when the sun is down
  – Distance to recreation doesn’t matter – who has time?
  – Distance to work, shopping, and parking matter a lot
• Joe likes to throw big parties
  – Interior aesthetics are very important
  – Joe is not fond of noise restrictions
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Stan the Family Man

• Stan goes to work to put in his time and get home to his family
• Stan’s wife wants a nice place with lots of recreation for the kids… Stan wants something he can afford
• The apartment needs to be kid-friendly
• Stan likes to drive – it gives him some quiet time
• Stan’s family is in bed by 10:00
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High Principles

• Modular
  – A decision is made up of atomic pieces of logic, called “considerations”
  – We can easily add and remove considerations

• Extensible
  – We can easily create new types of considerations

• Reusable
  – From decision to decision
  – From project to project
Terminology & Architecture

- **Reasoner** has a list of possible **choices**
  - E.G. play with a ball, build a swordsman unit, select a particular weapon, play a particular animation, etc.

- Each choice has a list of **considerations**
  - Considerations evaluate one aspect of the situation

- Considerations generate **appraisals**

- Appraisals inform our final selection
Consideration

• Encapsulates one aspect of a larger decision
  – Distance
  – Cost
  – Etc.
  – Selection History
  – Benefit

• Parameterized for easy customization
  – For this decision and for this character
class IConsideration
{
    public:
        // Load the data that controls our decisions
        void Load(const DataNode& node) = 0;

        // Evaluate this consideration
        Appraisal Evaluate(const Context& c) = 0;
}

Relies on:
    – DataNode
    – Context
    – Appraisal
DataNode

- XML (or equivalent) that contains the parameterization
- May be tool-generated (not hand-generated)
- May be part of a larger AI specification
Context

• Contains all of the information the AI needs to make a decision
• Provides an abstraction layer between the AI and the game
  – If well implemented, can facilitate porting your considerations from game to game
Appraisal

• Generated by the Evaluate() function
• Drives our final decision
• Common techniques include:
  – Boolean logic (e.g. all appraisals must return TRUE)
  – Highest score
  – Weight-based random
  – Optimize resource allocation to maximize utility
• Experience has taught me to start as simple as possible, extend only when necessary
Simple Utility-Based Appraisals

- Each appraisal contains two components:
  - **Base Score**: a floating point indicating how good we think this choice is (based on our one consideration)
  - **Veto**: a Boolean allowing each consideration to prevent us from selecting the associated choice

- Calculating total utility for a choice:
  - If any consideration sets Veto to false, utility is 0
  - Otherwise, add all of the base scores together
Example: Weapon Selection

- “Tuning consideration” provides a base score
  - A tuning consideration always returns the values specified in data, regardless of the situation
- “Range consideration” can add utility or veto as needed
  - Pistols are better at short ranges, sniper rifles at long
- “Inertia consideration” adds utility to current choice
  - So we don’t change without a good reason
- “Random noise consideration” has a random base score
  - So we don’t always pick the same thing
- “Ammo consideration” checks if we have ammo
- “Indoors consideration” prevents grenade use indoors
- Select the weapon with the best total score
Appraisal With A Multiplier*

• Replace Veto parameter with a “Final Multiplier”
  – Add all base scores together, then multiply by each of the final multipliers
  – A multiplier of 0 is still a veto

• Allows you to scale utility more smoothly/cleanly
  – For example, scale sniper rifle utility at short range

• Other things you could add:
  – Exponents
  – Polynomials
  – Etc.

* (This is Kevin’s preferred approach.)
Multi-Utility Appraisals

• Add a Priority attribute to the appraisal
• When combining appraisals, take the max Priority
  – In other words, if one consideration sets the priority to be high, keep that priority
• Only consider choices with max priority
  – Allows you to say “If X is true, only consider this small set of options.”
  – For example, force the use of a melee weapon at short range, a ranged weapon at long range
Summary

- Modular
- Extensible
- Reusable
- Applicable to a wide variety of game genres and reasoner architectures
  - *Kohan 2: Kings of War* and *Axis & Allies*
  - Prototype dog AI
  - *Iron Man* boss AI
  - *Red Dead Redemption*
    - Weapon Selection
    - Dialog Selection
  - Event selection in *All Heroes Die*
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Questions?

Dave Mark
Intrinsic Algorithm LLC
Dave@IntrinsicAlgorithm.com
www.IntrinsicAlgorithm.com
(402) 208–7054
IADaveMark
on:
Yahoo – AIM
Skype – Twitter

Kevin Dill
Lockheed Martin
kdill4@gmail.com

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